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Analysis of an AFC System for an Automatic Search - Track Receiver

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SYLVANIA ELECTRIC PRODUCTS INC.

CONTENTS

CONTENTS -- Continued

DEFINITION OF SYMBOLS

Symbol	Description	Section where Defined or First Used
B	bandwidth	8.3.1
D	signal to noise ratio at the input to the frequency sensing circuit	6.1.4
E	constant voltage	8.2.2
e	instantaneous voltage	3.2.3
e_c	error voltage corresponding to a frequency set error	3.2.1
e_t	local oscillator tuning voltage	3.2.1
F	noise figure	6.1.4
f	frequency	3.2.3
f_c	desired difference between local oscillator frequency and signal frequency	3.1
f_o	local oscillator frequency	3.1
f_s	signal frequency	3.1
K	amplifier midband gain	6.7
N_z	number of times per second that a noise voltage can be expected to pass through zero	8.3.1
n	false alarm rate	4.1
R	frequency following capability of the system	4.2
R_o	minimum value of R	4.2
T	time interval following each received signal, during which frequency correction takes place	3.2.1
χ	frequency set error	3.2.1
χ_d	frequency set error that arises as a consequence of tracking a changing signal frequency	5.2
α	constant of proportionality relating the net change in local oscillator tuning voltage to the amplitude of the error voltage	3.2.4
β	slope of the local oscillator tuning curve $\left(\frac{df_o}{de_t}\right)$	3.2.1

DEFINITION OF SYMBOLS

Symbol	Description	Section where Defined or First Used
γ	ratio of error voltage to the corresponding frequency set error	3.2.3
σ^2	mean square noise voltage	8.3.1
τ	interval between frequency correction pulses into the frequency control circuit	4.2
τ_o	minimum value of τ	4.2

ANALYSIS OF AN AFC SYSTEM FOR AN AUTOMATIC SEARCH-TRACK RECEIVER

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1. ABSTRACT.

The general behavior of a sampled data automatic frequency control loop for use in receiving pulsed radio frequency signals is described. A functional description of the system blocks is used to develop automatic frequency control (AFC) loop stability criteria. Tracking capabilities of the AFC loop are formulated and equations are derived to permit an estimation of frequency set errors. Design considerations are discussed. The frequency to voltage converter and the frequency control circuit are discussed in some detail in the appendixes.

2. INTRODUCTION.

This report is concerned with AFC systems capable of searching for and tracking pulsed radio frequency signals. When pulse widths are large and repetition rates are high, idealized approximations of system components may often be made with little error. When narrow pulse widths at low repetition rates are encountered, these idealizations may no longer be valid. For example, transient response times of system components as well as system drift between received pulses may have to be considered. The design problem is further compounded when rigid specifications are placed on frequency tracking capability and frequency set errors.

In order to optimize a given system in terms of tracking capability and frequency set errors, while maintaining AFC loop stability over all specified operating conditions, the nonlinear nature of the transfer functions must be taken into account. Since the system specifications are usually given in terms of maxima and minima, design requirements for system components may be generated in terms of the pertinent maxima and minima of the transfer functions (e.g., maximum slopes

2. -- Continued.

or maximum absolute values). Once the design is fixed, piecewise linear or graphical analysis techniques may be used to determine system performance under a given set of operating conditions.

A block diagram of the AFC system to be analyzed here is shown in Figure 1. Analysis of such a system employing a video intermediate frequency (IF) amplifier has been made using linear approximations of the component blocks of the AFC loop.[1,2]

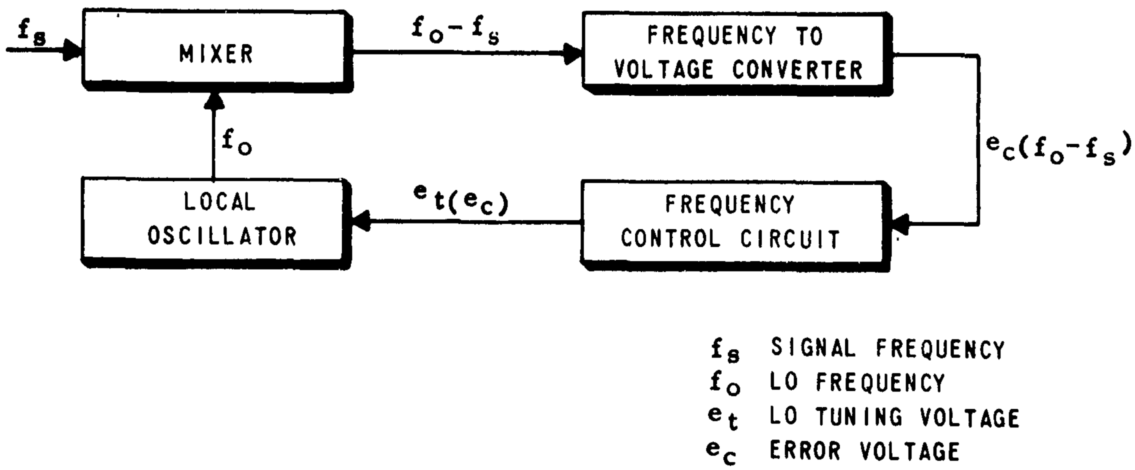

Figure 1

AFC System Block Diagram

The analysis to be presented here is for an AFC system with the following general characteristics:

(a) The system is to search for and track a pulsed received signal.

(b) The AFC system may be described by the block diagram of Figure 1.

2. -- Continued.

(c) The change in local oscillator (LO) frequency for each received pulse is related only to the instantaneous frequency error, not to any of its time derivatives.

An attempt will be made to separate the capabilities and limitations that are characteristic of the basic AFC loop configuration from those that depend on the details of the circuits used to realize certain functions. Such details as intermediate frequency or the type of frequency sensing circuit would fall in the last category. In order to extract the most meaningful design information, provisions will be made for quantitative inclusion of nonlinear transfer functions in the system analysis. Stability criteria, tracking capability, and frequency set errors will be formulated.

3. GENERAL AFC LOOP DESCRIPTION.

3.1 Basic Operation.

The frequency control circuit provides the dual functions of tuning the LO of frequency f_o through the desired frequency range during the search phase and providing frequency correction voltages when the system is tracking a pulsed signal of frequency f_s. In the absence of a signal or when the magnitude of the difference frequency $(f_o - f_s)$ is outside the bandpass of the frequency-to-voltage converter, the frequency control circuit generates a sawtooth tuning voltage that tunes the LO through the desired frequency range. Assuming that a signal is within this frequency range, the difference frequency $(f_o - f_s)$ will, at some time during the sweep, fall within the frequency-to-voltage converter bandpass. Upon receipt of a signal within its bandpass, the frequency-to-voltage converter will provide two functions. The first function is to trigger logic circuits that change the frequency control circuit's operation from that of a sweep generator to an integrating amplifier and the second function is to generate pulses of the proper polarities and amplitudes to adjust the frequency control circuit output, consequently the LO frequency, so that the difference frequency $(f_o - f_s)$ approaches a prescribed value f_c.

3.2 Functional Description of the System Blocks.

3.2.1 Local Oscillator. The frequency characteristic for the local oscillator is assumed to be a continuous monotonic increasing (or monotonic decreasing) curve, i.e., corresponding to each frequency in the range of interest, there is a single value of voltage, and conversely, for each value of voltage, there is a unique frequency. Such a curve is illustrated in Figure 2. The slope of the curve at any point $\frac{df_o}{de_t}$ is defined as β.

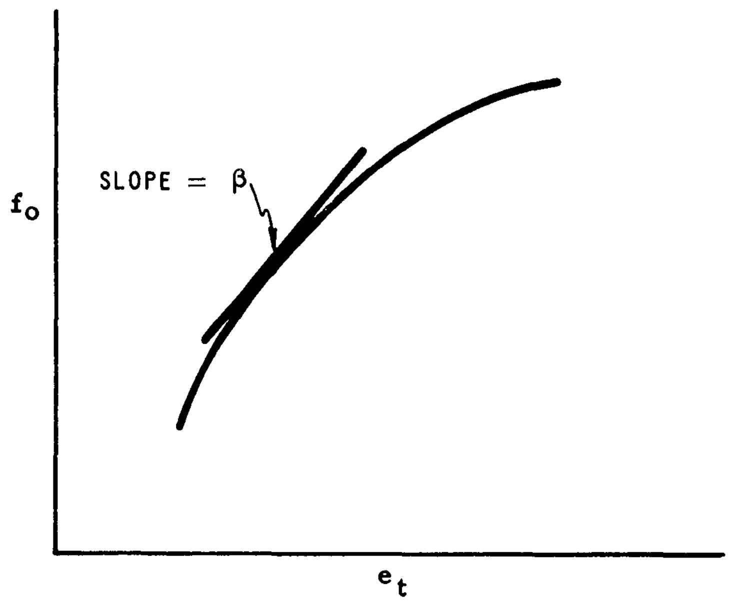

Figure 2

Local Oscillator Tuning Curve

3.2.2 Mixer. The mixer is assumed to reproduce the received pulses at the difference frequency $f_o - f_s$, i.e., the carrier is altered by an amount f_o.

3.2.3 Frequency-to-Voltage Converter. The frequency-to-voltage-converter contains an IF (or video) amplifier-limiter, a frequency sensing circuit, a pulse stretcher, and a pulse amplifier.

3.2.3 -- Continued.

The amplifier-limiter is necessary for two reasons. First, it must amplify the mixer output pulses to a usable level consistent with accurate operation of the circuits to follow. Second, since a wide range of signal amplitudes may be expected, limiting is necessary to reduce or eliminate the effects of signal amplitude on the frequency-to-voltage converter output. The two amplifier configurations to be considered here are a finite (60 megacycle) IF amplifier and a "zero IF" video amplifier. Since the pulses at the output of the amplifier-limiter will have a nearly constant amplitude, the shape of the frequency-to-voltage converter transfer curve will be determined essentially by the frequency sensing element.

To illustrate how frequency-to-voltage conversion may be obtained, consider the amplitude versus frequency curve of a video amplifier, as shown in Figure 3. The RF pulses from this amplifier are detected and the peak amplitudes are stored in a pulse stretcher. The AFC loop is to be designed to set the LO at an offset frequency of f_c; i.e., the frequency set error is defined to be zero when the difference frequency $(f_o - f_s)$ equals f_c. If pulses of amplitude e_o are subtracted from the output of the pulse stretcher, the curve of frequency set

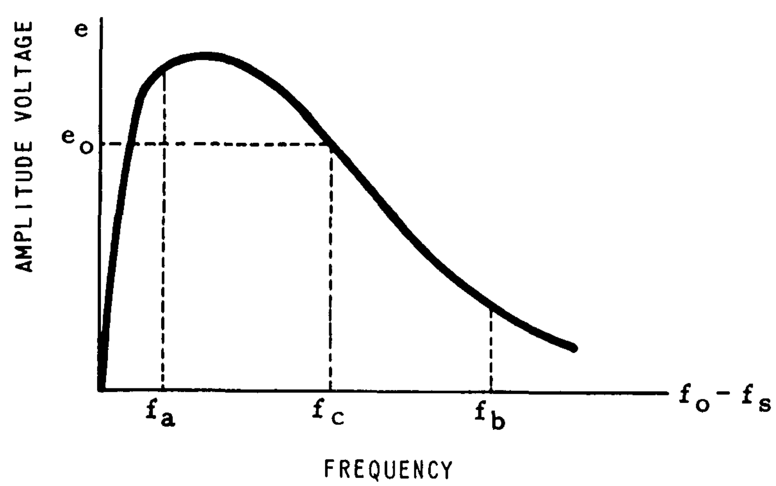

Figure 3

Amplitude-versus-Frequency Curve of a Video Amplifier

3.2.3 -- Continued.

error as a function of error voltage would be that of Figure 3, with the origin of coordinates shifted to (f_c, e_o). The pass band of the IF (or video) amplifier would limit the usable portion of the curve of Figure 3 to the range between f_a and f_b. The frequency sensing element described above will hereafter be called a slope detector.

A second method of achieving frequency-to-voltage conversion is with a discriminator in which the outputs of two amplifiers with different center frequencies are subtracted to obtain the familiar "S-curve". The offset frequency (f_c) for a discriminator would be the crossover frequency of the S-curve. For a more detailed discussion of the frequency sensing circuits, see Appendix 8.1.

The function of the pulse stretcher is to reproduce the information (pulse amplitude) from the frequency sensing circuit in the form of wide pulses. The principal reason for stretching is to lower the frequency response requirements of the circuits that follow. The limit on the amount of stretching that may be used without losing information of course depends on the duty factor of the received signal.

The transfer function of the frequency-to-voltage converter (including amplifier-limiter, frequency sensing circuit, pulse stretcher, and pulse amplifier) will be a curve such as one of those shown in Figure 4.

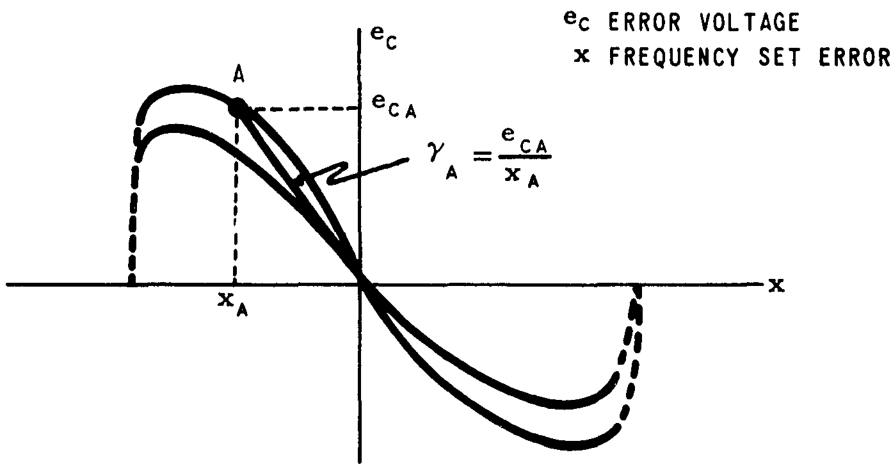

Figure 4

Frequency-to-Voltage Converter Transfer Function

3.2.3 -- Continued.

Here e_c is defined as the amplitude of the pulse out.* The variable x is defined as:

$$x = f_o - f_s - f_c \tag{1}$$

Hence, x is the frequency error or the amount that the frequency ($f_o - f_s$) differs from the desired offset f_c. Gamma (γ) is defined as the slope of a line drawn through the origin intersecting the curve, (At a point e_c, x where x does not equal zero.) equation 2.

$$\gamma = \frac{e_c}{x} \tag{2}$$

Since limiting will not be perfect and the frequency sensing circuit may respond differently for narrow and wide input pulses, a family of curves may be plotted, that depend on the signal input. The two curves shown in Figure 4 illustrate the curves of maximum and minimum slope at the origin.

3.2.4 Frequency Control Circuits. In the search or open loop mode of operation, the frequency control circuit generates a sawtooth voltage that is used to tune the LO through the desired frequency range. In the track or closed loop mode of operation, the tuning voltage and, consequently, the LO frequency are held constant until a pulse is received from the frequency-to-voltage converter. When a pulse is received, a correction interval T is provided, during which the tuning voltage and the LO frequency are corrected by an amount proportional to the amplitude of the received pulse. This new value of tuning voltage is then held until another pulse is received or sufficient time has elapsed to indicate loss of signal, and searching is resumed. A circuit configuration capable of performing the indicated functions is shown in Figure 5.

This frequency control circuit consists of a Miller integrator type of circuit, an amplifier, and the logical networks necessary to change the input to the Miller integrator upon receipt of a trigger from the frequency-to-voltage converter. This switching function is shown

* For a system that is not zero seeking, i.e., a system in which a finite error voltage e_{co} would correspond to zero frequency set error, e_c should be replaced by $-e_{co}$ in all of the following derivations.

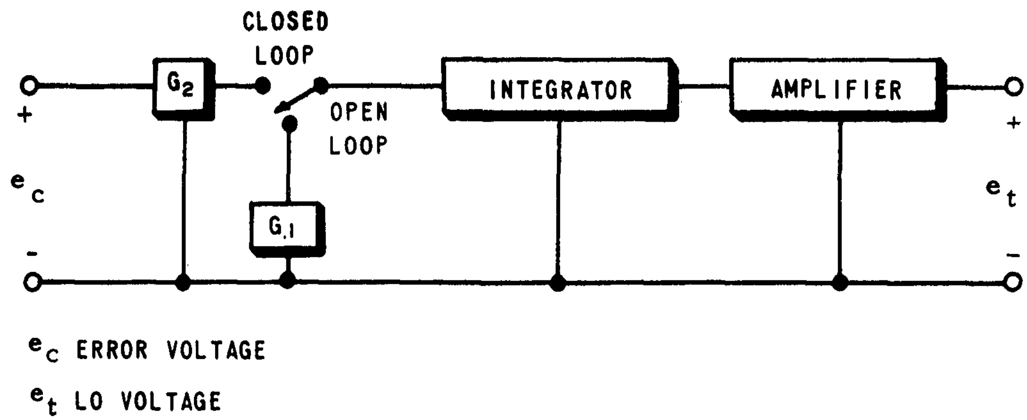

Figure 5

Frequency Control Circuit

3.2.4 -- Continued.

symbolically by a SPDT switch. During open loop operation, the circuit is a free-running sawtooth generator. During closed loop operation, the net change in e_t per pulse is given by

$$\Delta e_t = \alpha e_c \tag{3}$$

where α is a constant depending on the particular circuit configuration. For the circuit details, as well as the derivation of Equation 3, see Appendix 8.2.

4. AFC LOOP STABILITY AND TRACKING.

4.1 Stability.

Assume that the system is in closed-loop operation, i.e., the difference between the signal frequency and the LO frequency is within the AFC bandwidth. In addition, assume that a pulse has just been received

4.1 -- Continued.

and that the frequency error at this instant is x. (Both positive and negative errors are defined, depending on whether the difference frequency is above or below the desired offset.) Before the next pulse is received, a stable system must adjust, so that the magnitude of the error at the instant the next pulse is received is smaller than the magnitude of the original error, x. Putting this statement into mathematical form will lead to the stability criterion. Corresponding to the frequency error, x, the frequency-to-voltage converter output will be a pulse of amplitude e_c (Figure 4). The change in LO tuning voltage Δe_t due to this pulse is given by Equation 3, and is equal to αe_c. This change in Δe_t (see Figure 2) will in turn change the LO frequency by

$$\Delta f_o = \Delta x = \Delta e_t \frac{df_o}{de_t} = \alpha \beta e_c . \quad (4)$$

Now, to reduce the error,

$$| x + \Delta x | < | x | , \quad (5)$$

or

$$x^2 + 2x \Delta x + \Delta x^2 < x^2 . \quad (6)$$

This may be reduced to

$$\frac{2x}{\Delta x} < -1 . \quad (7)$$

An alternative statement is

$$0 > \frac{\Delta x}{x} > -2 . \quad (8)$$

Now, substituting the value of Δx given by Equation 4,

$$0 > \frac{\alpha \beta e_c}{x} > -2 . \quad (9)$$

Substituting Equation 2 into Equation 9, we have for stability

$$0 > \alpha \beta \gamma > -2, \quad (10)$$

for all values of α, β and γ.

4.1 -- Continued.

For $\alpha \beta \gamma = -1$, the loop is critically damped and a single pulse will reduce the error to zero. Sketches of critically damped, over-damped ($0 > \alpha \beta \gamma > -1$), and under-damped ($-1 > \alpha \beta \gamma > -2$) responses are shown in Figure 6 for an unvarying received signal frequency.

The frequency control circuit may be designed so that α is very nearly constant. The LO tuning characteristic will generally be nonlinear, so β will depend on the tuning voltage. Although the frequency-to-voltage converter characteristic may be made quite linear in the neighborhood of the origin, it will generally be nonlinear at the extremities. In addition, since perfect limiting cannot be achieved and rise times will be finite, the curve of Figure 4, e_c as a function of x, will be rotated and also possibly shifted with changing signal amplitudes and pulse widths. Hence, γ will depend on the signal characteristics as well as on the frequency error. Thus, the loop must be designed to be stable with the extreme values of β and γ.

For the curves shown (Figures 2 and 4), β is positive and γ is negative. Hence α must be positive and for stability

$$0 < \alpha \beta_{max} |\gamma|_{max} < 2 \quad . \tag{11}$$

It will be shown that the optimum signal tracking capability would be achieved for a linear system (α, β, γ, all constant) with

$$\alpha \beta \gamma = -1 \tag{12}$$

With β and γ nonlinear, a stable system with a reasonable high tracking capability may be designed by taking

$$|\alpha| |\beta|_{max} |\gamma|_{max} = 1 \tag{13}$$

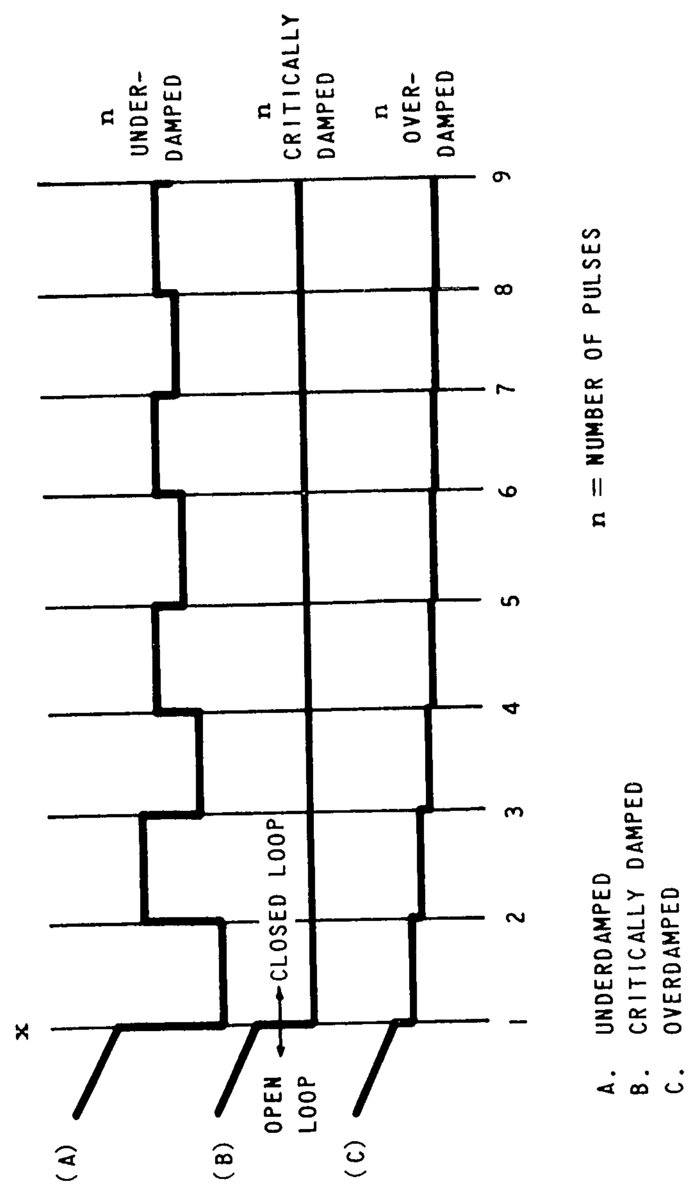

Figure 6
AFC Loop Response

4.2 Tracking.

Consider a signal frequency changing at a constant rate R megacycles per second per second. If $1/\tau_o$ is the minimum information rate (τ_o is the maximum time between pulses into the frequency control circuit), the maximum signal frequency change between correction pulses is $R\ \tau_o$. The tracking capability of the system will be defined as that value of R at which the system will just keep the difference frequency within the bandpass of the frequency-to-voltage converter. This capability will be evaluated under the worst conditions, i.e., maximum τ lowest magnitude of β, etc. This will be the minimum tracking capability of the system, and will be written as R_o (Mc/sec^2). For a signal changing at a constant rate of R (Mc/sec^2), and AFC loop frequency correction per pulse must be

$$\Delta x = R\tau = \alpha \beta e_c . \tag{14}$$

The limiting value of $R\tau$ that the AFC system can follow corresponds to the peak correction per pulse. Thus, the minimum tracking capability is given by

$$R_o \tau_o = |\alpha| \, |\beta|\, \text{min} \, |e_c \text{ peak}| \tag{15}$$

where $|e_c \text{ peak}|$ is the value at the peak of the transfer function curve for the frequency-to-voltage converter. For an unsymmetrical curve, it will be the value at the peak of lower magnitude.

For example, where α and β are positive, γ negative, and with

$$\alpha \beta_{max} \, |\gamma|_{max} = 1 \quad , \tag{16}$$

$$R_o \tau_o = \frac{\beta \text{ min}}{\beta \text{ max}} \quad \frac{|e_c \text{ peak}|}{|\gamma|_{max}} \quad , \tag{17}$$

Usually, the peak of lower magnitude of the transfer function curve will correspond to the minimum magnitude of γ. When this is the case, Equation 17 is subject to a simple physical interpretation. Substituting

$$|e_c \text{ peak}| = |\gamma|_{min} \, |x_{peak}| \tag{18}$$

4.2 -- Continued.

into (17) we have

$$R_o \tau_o = \frac{\beta \min}{\beta \max} \quad \frac{|\gamma| \min}{|\gamma| \max} \quad |x \text{ peak}| \tag{19}$$

$|x \text{ peak}|$ is the maximum amount by which the signal frequency may change between pulses without causing the system to lose track. Hence, the AFC bandwidth is the frequency difference between the peaks of the transfer function curve. The absolute value of x_{peak} is the value of $R_o \tau_o$ for an ideal, linear, critically damped system. The ratios of β_{min} to β_{max} and $|\gamma|_{min}$ to $|\gamma|_{max}$ may be thought of as degradation factors, arising from nonlinear transfer functions, that reduce the minimum tracking capability.

Note that the ratio $|\gamma|_{min}$ to $|\gamma|_{max}$ must be evaluated under the extremes of signal input. It is probable that $|\gamma|_{min}$ would be the minimum ratio of e_c/x with $|e_c| \leq |e_{c\ peak}|$ for the transfer function curve which is plotted under the conditions of minimum specified signal amplitude and minimum pulse width. Conversely, $|\gamma|_{max}$ would probably be the maximum ratio of e_c/x with maximum signal amplitude and pulse width.

5. FREQUENCY SET ERRORS.

5.1 Static Errors.

The total static error will be defined as the linear combination of the expected errors in frequency set for an unvarying signal frequency. The only sources of errors considered here are those sources which provide appreciable error during the pulse period τ, e.g., errors due to frequency modulation of LO would be considered, whereas thermal drift of the LO would be neglected.

One static error introduced by the frequency-to-voltage converter will be a change in the zero crossover points of the transfer function curve. The change in the zero crossover point may result from changes in the signal level, pulse width, or PRF of the signal. Any estimate of the change in the zero crossover point will depend upon the circuits used to realize the frequency-to-voltage converter function. An estimate of

5.1 -- Continued.

the zero crossover change due to changes in signal pulse width is made for two particular circuits in the Appendix, Sections 8.1.1 and 8.1.2 (see Equations 49 and 62).

The static error due to a finite signal-to-noise ratio in the frequency-to-voltage converter circuits is often very small, whenever a tolerable false alarm is specified (see Appendix 8.3).

The frequency control circuit will generally introduce two types of static errors. The errors introduced into the AFC system arise from the dual functions of the frequency control circuit, one an open loop (search) mode, and the second a closed loop (track) mode. If the switching used to provide the dual modes is not perfect, static frequency errors will be generated.

When electronic switches are used, the switches have a threshold voltage that must be overcome before the electronic switch acts as a closed circuit. In cases where the switch exhibits this behavior, the static error due to the threshold voltage may be calculated by dividing the threshold voltage (see Equation 84) by the slope of the frequency-to-voltage converter transfer function curve at the origin (γ_o). (See Equations 53 and 64.)

A second error due to electronic switches occurs when the switch has a leakage current. Calculations of this error may be made quite simply by multiplying the change in the tuning voltage, e_t, by the maximum slope, β_{max}, of the local oscillator tuning curve. (See Equation 86).

The second type of static error introduced by the frequency control circuit arises from hum and noise generated within the circuit. Any noise or hum generated within the frequency control circuit will cause frequency modulation of the LO.

Static errors may also be introduced by any changes in the LO frequency not related to the tuning voltage. Voltage tuned oscillators employing more than one voltage for operation will generally be susceptible to frequency changes due to changes in electrode voltages.

5.2 Dynamic Error.

The dynamic error will be a measure of the frequency set error that arises as a consequence of tracking a signal whose frequency is changing.

Consider a signal whose frequency is changing at an average rate of R megacycles per second per second. Suppose that the time between received pulses is τ. The signal frequency is illustrated by the set of discrete points in Figure 7. The dashed line connecting these points has a slope of $R\,(\mathrm{Mc/sec}^2)$. The AFC system receives a correction pulse at time t_1 and the LO frequency, shown by the serrated line, is corrected for a time T ($T < \tau$), and then remains constant until another pulse is received at $t = t_1 + \tau$. If random frequency perturbations within the loop and frequency drift between pulses are negligible, the steady-state error at the time of receipt of a pulse will be constant at the value $f_{o1} - f_{s1} - f_c$. This error at the time of receipt of the correction signal will be defined as the dynamic error x_d.

If the exact shape of the frequency-to-voltage converter transfer function curve is known, an accurate estimate of x_d may be made quite simply. Recall from the section on tracking (Equation 14) that the frequency correction per pulse is

$$\Delta x = \alpha \beta e_c \quad . \tag{20}$$

But for the steady-state condition, the correction per pulse must equal the signal frequency change between pulses.

Thus

$$\Delta x = R\tau = \alpha \beta e_c \tag{21}$$

Solving for the steady state value of e_c,

$$e_c = \frac{R\tau}{\alpha\beta} \quad . \tag{22}$$

Knowing e_c, one may read x_d directly from the frequency-to-voltage converter transfer function curve.

To establish bounds on the dynamic error x_d for $|R\tau| < |R_o \tau_o|$, ($R_o \tau_o$ defined in Section 4.2, Equation 15) suppose the limiting values of $|\gamma|$ are γ_1 and γ_2 for $|e_c| \leq |e_{c\ \mathrm{peak}}|$

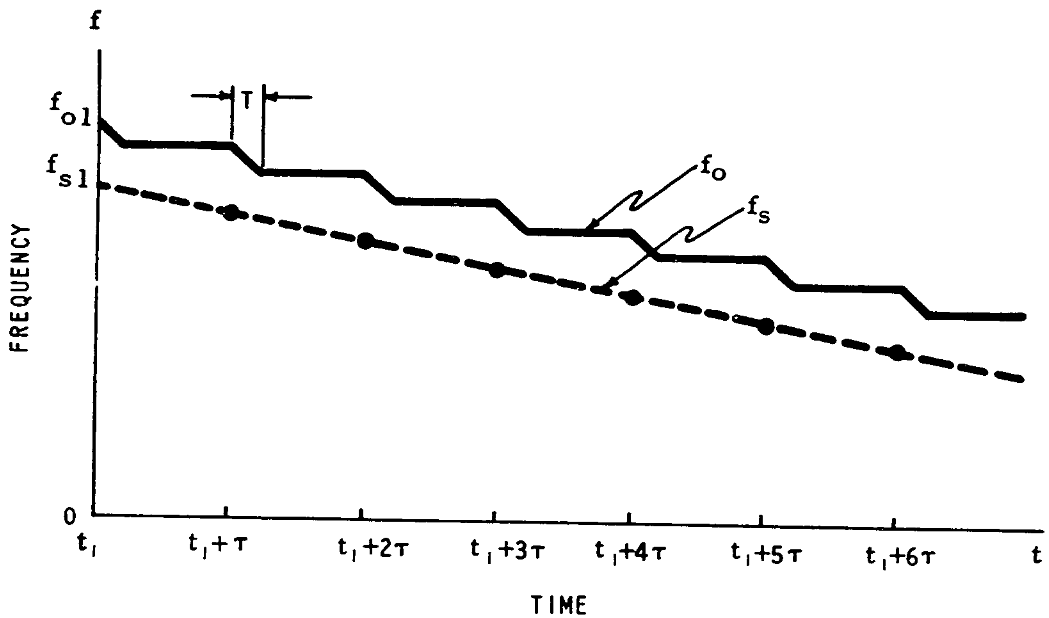

Figure 7

Local Oscillator Response for a Varying Signal Frequency

5.2 -- Continued.

If
$$\gamma_1 \le |\gamma| \le \gamma_2 \tag{23}$$

then
$$\frac{1}{\gamma_2} \le \left| \frac{x_d}{e_c} \right| \le \frac{1}{\gamma_1} \tag{24}$$

since
$$\gamma = \frac{e_c}{x} \quad . \tag{25}$$

The net frequency change per pulse Δx is given by

$$\Delta x = R\tau = \alpha\beta e_c \tag{26}$$

Thus,

$$\left| \frac{R\tau}{\alpha\beta\gamma_2} \right| \le | x_d | \le \left| \frac{R\tau}{\alpha\beta\gamma_1} \right| \tag{27}$$

Note that for the ideal system, where β and γ are constant, and

$$|\alpha\beta\gamma| = 1, \tag{28}$$

$$x_d = R\tau \quad , \tag{29}$$

as expected.

6. DESIGN CONSIDERATIONS.

In many applications, the received signal characteristics will determine the system characteristics as to maximum information rate, pulse width, sensitivity, and tracking. In other applications, such as slaving one oscillator to another, where the signal characteristics are at the disposal of the designer, the generation of specifications on information rate, pulse width, and signal level will be governed by the specifications on tracking capability and frequency set accuracy.

6.1 System Specifications.

A number of possible performance requirements that might be specified are listed below. The specifications for a particular system would probably include several items from this list, not necessarily all of them. In fact, some of these specifications may be mutually exclusive under certain conditions or, at best, redundant. For example, the AFC bandwidth affects both the signal tracking capability and the ability of the system to function properly with the minimum signal pulse width. For any list of specifications, including both signal tracking capability and minimum pulse width, one or the other would probably be dominant in fixing the AFC bandwidth. The interdependence between these specifications will be pointed out in the sections to follow.

6.1.1 Information Rate. The minimum information rate is defined as the minimum rate at which frequency correction pulses are fed to the frequency control circuit. Obviously, it can be no greater than the minimum PRF of the received signal. Where look-through is used (periodic sampling of the input), it may be considerably lower. The minimum information rate is important in determining minimum tracking capability and maximum search rate.

The maximum information rate is important in specifying recovery times for the individual circuits in the loop. If the loss of frequency correction information is to be avoided, all transients following one correction pulse should be settled out before the next pulse is received.

6.1.2 Pulse Width. The range of anticipated signal pulse widths will generally be specified. The specification of greatest interest here is that of minimum pulse width, since the bandwidths of the circuits up to and including the pulse stretchers must be adequate to accommodate the narrowest pulses.

6.1.3 Tracking Capability. The minimum tracking capability is determined by the minimum information rate, linearity and bandwidth of the frequency-to-voltage converter, and linearity of the LO tuning curve (see Equation 19). Thus, if the shapes of the LO tuning curve and the frequency-to-voltage converter transfer function curve are fixed, as well as the minimum information rate, either minimum pulse width or minimum tracking capability may be the dominant factor in determining the necessary frequency-to-voltage converter bandwidth.

6.1.4 Search Rate. The maximum rate at which the LO may be swept while maintaining a 100 per cent probability of intercept of a signal within its range is governed by the minimum information rate and the frequency-to-voltage converter bandwidth. The maximum LO frequency change between frequency correction pulses must be less than the frequency separation between the peaks of the frequency-to-voltage converter transfer function ($f_b - f_a$). If this is evaluated at the minimum information rate ($1/\tau_o$), we have:

$$\left| \frac{d f_o}{d t} \right|_{max} < \frac{f_b - f_a}{\tau_o} \qquad (30)$$

6.1.5 Static Error. Static error is defined as the average magnitude of the frequency set error with an unvarying signal frequency. Although it is a very reasonable quantity to specify, it is a difficult specification to include quantitatively in any design because of the number of sources of errors. Some of these sources are discussed briefly in Section 5.1. If the designer places maximum limits on the separate error contributions, design limits for individual components may be generated (e.g., a specification of maximum frequency drift due to imperfect switching will lead to a specification of maximum tolerable reverse leakage current for an electronic switch).

6.1.6 Sensitivity. The maximum sensitivity of the AFC loop may be defined as the minimum signal level that will cause the AFC loop to operate within the error and tracking specifications. A tangential or 2:1 S/N ratio definition has no real meaning for this application.

Selection of the amplifier A_o bandwidth B_o on the basis of the specification for pulse width or on the basis of a specified tracking rate, will determine the bandwidth B_o. Normalizing the sensitivity S to a one megacycle bandwidth, i.e., KTB at standard conditions for a one-megacycle bandwidth the sensitivity in dbm will be:

$$S \geq -114 + 10 \log 2B_o + D + F \qquad (31)$$

where F is the noise figure, in db, of the mixer, local oscillator and amplifier, A_o.

6.1.6 -- Continued.

D is the signal-to-noise ratio in db necessary for the system false alarm rate as described in Section 8.3.1 and B_o is the amplifier bandwidth.

Any specification for high sensitivity will require that bandwidth must be conserved and that every attempt be made to reduce the noise figure of the mixer, local oscillator and amplifier.

6.2 Local Oscillator.

The LO tuning characteristic was discussed briefly in Section 3.2.1. The requirement of a monotonic increasing (or monotonic decreasing) curve was subsequently shown to be necessary for stability; that is, for a given frequency set error, the incremental change in LO frequency must be in such a direction as to decrease that error. Equation 19 illustrates the fact that the curve should be as linear as possible in order to achieve a high tracking capability. If the ratio of β_{min} to β_{max} for the tube itself is very much less than unity, a passive linearizer network may be added at the input to the LO so that the ratio of β_{min} to β_{max} as seen at the input to this network is improved. Linearization networks are quite common and will not be covered here.

The linearization mentioned will compensate only for changes in gross slope. There may also be local variations in slope (fine structure). As long as the slope remains finite and does not change in sign, the loop may be designed to be stable. These local variations in slope will, however, degrade the tracking capability if they take place over a frequency range comparable to the AFC bandwidth.

6.3 Mixer.

In systems employing a zero IF amplifier and a slope detector, it is necessary to employ a mixer which will provide rejection of the envelope detection of the RF signal. In these systems, the detected envelope of the RF signal may be rejected by employing a balanced mixer using crystals of opposite polarity in the separate halves of the mixer. The balanced mixer while rejecting the detected envelope will also provide appreciable rejection of the LO noise.

6.3-- Mixer.

In systems using a finite IF amplifier, rejection of the detected envelope is not necessary, but a balanced mixer may be employed for the LO noise rejection characteristics.

6.4 Frequency-to- Voltage Converter.

The frequency-to-voltage converter characteristics will, for a large part, determine the system sensitivity, the maximum tracking capability and the minimum frequency set errors. The over-all characteristics of the frequency-to-voltage converter may be best discussed by examining the characteristics of the blocks within the frequency-to-voltage converter.

Important considerations in the specification of the characteristics of amplifier A_o are rise time of the pulsed signal, limiting to reduce the effective dynamic signal variation, the amplifier noise figure, and the gain necessary to amplify the signal to a usable amplitude. The gain and noise figure of the frequency-to-voltage converter will be generally determined by the amplifier A_o.

The rise time of the amplifier A_o for a pulsed signal must be sufficient to allow the output signal to reach its final amplitude during the pulse interval. Since the rise time of an amplifier is not uniquely related to its bandwidth, while amplifier noise is directly related to bandwidth, the optimization of rise time and noise figure of amplifier A_o will depend on the particular amplifier configuration, and, as such, will not be covered here. It should be recalled that the bandwidth of the frequency-to-voltage converter (likewise the bandwidth of A_o) must be sufficient for the minimum tracking capability (see Equation 19).

Limiting of the output signals of A_o for changing levels of the input from the mixer is necessary to reduce the variation of the transfer function curve due to signal level changes. Any variation of the curve with signal level will reduce the minimum tracking capability.

The important characteristics of the frequency sensing circuits, A_1 and A_2, are rise time for a pulsed signal and controlled bandpass characteristics. A thorough discussion of these and other characteristics is presented in the Appendix, Section 8.1.1.

6.4 -- Continued.

The rise time and pulse droop characteristics of the pulse stretcher circuits play an important role in determining that the frequency-to-voltage converter will have a unique zero. Any difference in the rise times of the pulse stretcher circuits may cause transients in the output of the difference amplifier at the beginning of the correction period.

The pulse droop of the stretched pulses should be small and nearly identical. Any difference in the droop characteristics will cause the difference amplifier output to change during the correction interval.

The difference amplifier should provide a stable gain since changes in gain will cause changes in the transfer function curve of the converter. Since the rise times of the outputs from the pulse stretchers may not be identical, the transient at the output of the difference amplifier may be reduced by reducing the frequency response of the difference amplifier.

6.5 Frequency Control Circuit.

The specifications and characteristics of the frequency control circuit have been covered in detail in the Appendix, Section 8.2.

6.6 Tuning Control Amplifier.

Generally a tuning control amplifier will be needed to amplify the voltage from the frequency control circuit to a level suitable for the control of the LO frequency (see Figure 13). The tuning control amplifier should have a low noise level and a stable gain.

The frequency response of the tuning control amplifier may be specified by determining a suitable transient response. If high tracking capabilities are to be maintained at the highest information rate, the transients caused by correction in the frequency control circuit voltage should be reduced to a negligible level before the next information is received into the AFC system.

Stable gain and low noise characteristics may be accomplished by using degenerative feedback. Particular emphasis should be placed upon the circuits to reduce the hum from the heaters, power supplies, etc., as hum and noise from the tuning control amplifier will introduce frequency set errors into the AFC system.

6.7 Example.

A detailed design example would be too lengthy to include in this report. However, some brief sample calculations will be carried out for the typical specifications listed below:

Pulse width	0.1 to 5.0 μsec
Information rate	100 to 2000 pps
Minimum tracking capability	400 Mc/sec^2
Sensitivity	-66 dbm
Dynamic signal variation	+30 db

Differences between the zero IF and finite IF systems will be pointed out where they exist.

Suppose a system is to be designed to control the frequency of a local oscillator over a range of 3000 Mc. Assume that the LO slope changes from 5 megacycles per volt to 7.5 megacycles per volt as it is tuned through this range.

An iterative procedure would probably be used for the design of any particular system. This would involve estimating the performance capabilities of some circuits in order to fix the design constraints for other circuits. Then, the individual circuits may be analyzed or built and tested to provide data for a more accurate design.

For the problem at hand, we may begin by estimating the AFC bandwidth required to meet the specification on minimum tracking capability. The specified values of R_o and τ_o are 400 megacycles per second per second and .01 second, respectively. The ratio of β_{min} to β_{max} for the LO is 2/3. The dynamic range of signal variation is 30 db. Reduction of this variation by limiting to an effective dynamic range of 3 db should be possible. Since variation of the amplitude of the signal into the frequency sensing circuits causes rotation of the frequency-to-voltage converter transfer function curve about the origin, the ratio of γ_{min} to γ_{max} will be about 2/3. Thus, we have from Equation 19

$$|x_{peak}| = 9 \text{ Mc} \qquad (32)$$

6.7 -- Continued.

Since the AFC bandwidth is the frequency difference between the peaks of the frequency-to-voltage converter transfer function curve, the AFC bandwidth required is about 18 megacycles. This bandwidth should be more than adequate to accommodate pulses of the minimum specified pulse width. From the standpoint of noise bandwidth and reducing the required number of amplifier stages, it would probably be desirable to reduce this bandwidth. Some reduction may be accomplished by linearizing the LO tuning curve. Achieving a range of slopes from 5 megacycles per volt to 6 megacycles per volt for the LO plus linearizer should not be too difficult. Increasing the ratio of β_{min} to β_{max} from 2/3 to 5/6 reduces the required AFC bandwidth to 15 Mc.

Performance requirements for the circuits of the frequency to voltage converter may now be detailed. The particular configuration shown below is discussed in detail in Appendix, Section 8.1. Referring to Figure 8, the response of amplifier A_o should be flat over a range of 15 megacycles for the finite IF (60-megacycle) system and out to 20 megacycles for the zero IF system. These values are chosen to give equivalent AFC bandwidths of 15 megacycles for the two systems. The region from 0 to 5 megacycles is not useful for the zero IF amplifier since a difference frequency greater than this is required to give at least one half cycle of output with 0.1 microsecond pulses.

The frequency sensing circuits will be different for the finite IF and the zero IF configurations. For a zero IF and a slope detector, assume that the amplitude versus frequency curve for amplifier A_1 has a simple one-pole roll off. The gain of A_1 normalized to unity is:

$$K_{1n} = \frac{1}{\sqrt{1 + (f/B_1)^2}} \qquad (33)$$

where f is the difference frequency (f_o - f_s) and B_1 is the frequency at the upper 3 db point. Amplifier A_2 will be wide-band (no appreciable change in gain up to 20 Mc). It is shown in Section 8.1.2 that the optimum value of the gain of A_2 (using the same normalization factor) is

$$K_{2n} = \frac{1}{\sqrt{2}} \qquad (34)$$

Thus, the output of the frequency-to-voltage converter is of the form:

$$e_c = Ke_{in} \left\{ \frac{1}{\sqrt{1 + (f/B_1)^2}} - \frac{1}{\sqrt{2}} \right\} \qquad (35)$$

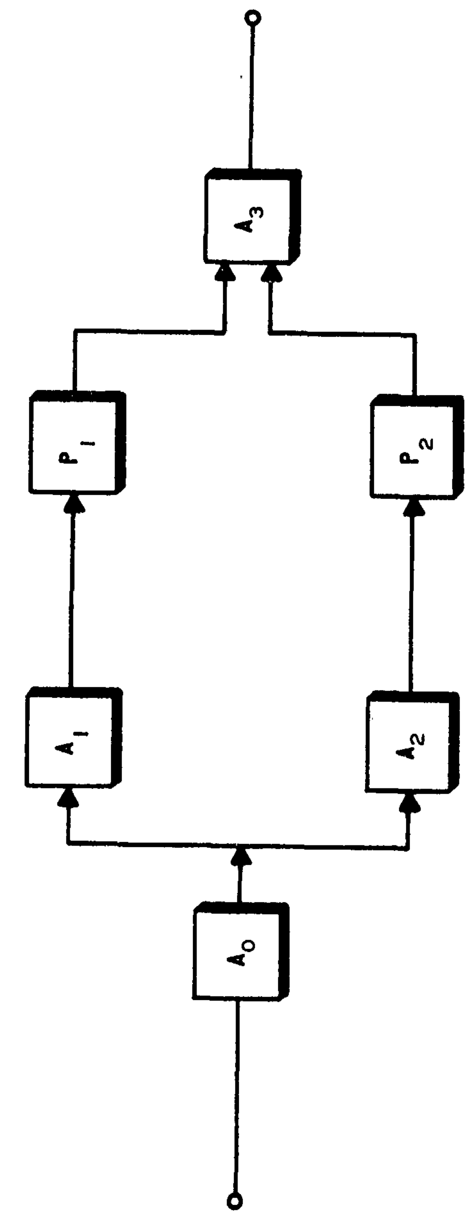

Figure 8
Frequency-to-Voltage Converter

6.7 -- Continued.

Taking B_1 = 12.5 gives an offset frequency f_c of 12.5 megacycles (f_c is the frequency at which $e_c = 0$) and a control range of 5 to 20 megacycles. A plot of e_c as a function of the frequency error x, normalized to $e_c = 1$ at x = -7.5 megacycles, is shown in Figure 9. The ratio of $\gamma_{min}/\gamma_{max}$ for this curve is 0.8. However, this is but one of a family of curves that will be produced as the signal is varied. The minimum value of $\gamma_{min}/\gamma_{max}$ under the extremes of input signal level must be greater than 0.64 in order that the specification on tracking capability be met with an AFC bandwidth of 15 megacycles. Thus, the change in slope (rotation) of the curve over the entire range of input signal level must be less than 1.25:1. This corresponds to a change in signal level after limiting of 2 db.

For a finite IF and a discriminator, amplifiers A_1 and A_2 will be assumed to be single-tuned stages. The gain of A_1 normalized to unity at its center frequency is

$$K_{in} = \frac{1}{\sqrt{1 + \left(\frac{f^2 - f_1^{\,2}}{f B_1}\right)}} \tag{36}$$

where f_1 is the center frequency for A_1 and B_1 is the frequency difference between the 3 db points of A_1. A similar expression may be written for K_{2n}. The output of the frequency-to-voltage converter for this system is of the form

$$e_c = Ke_{in} \frac{1}{\sqrt{1 + \left(\frac{f^2 - f_1^{\,2}}{f B_1}\right)^2}} \; \frac{1}{\sqrt{1 + \left(\frac{f_2^{\,2} - f^2}{f B_2}\right)^2}} \tag{37}$$

Taking B_1 and B_2 at 20 megacycles, f_1 at 48 megacycles, and f_2 as 70 megacycles gives an offset frequency f_c of 60 megacycles and a fairly linear transfer function curve, e_c to x for the values of x between -7.5 and +7.5 megacycles. ($x = f - f_c$). A plot of the curve normalized to e_c equal to unity at x equal to -7.5 megacycles is shown in Figure 10.

The ratio of $\gamma_{min} / \gamma_{max}$ for this curve is 0.9. Thus, the change in slope due to imperfect limiting must be less than 3 db in order that the specification on tracking capability be met with an AFC bandwidth of 15 Mc.

6.7 -- Continued.

Taking B_1 = 12.5 gives an offset frequency f_c of 12.5 megacycles (f_c is the frequency at which $e_c = 0$) and a control range of 5 to 20 megacycles. A plot of e_c as a function of the frequency error x, normalized to $e_c = 1$ at x = -7.5 megacycles, is shown in Figure 9. The ratio of $\gamma_{min}/\gamma_{max}$ for this curve is 0.8. However, this is but one of a family of curves that will be produced as the signal is varied. The minimum value of $\gamma_{min}/\gamma_{max}$ under the extremes of input signal level must be greater than 0.64 in order that the specification on tracking capability be met with an AFC bandwidth of 15 megacycles. Thus, the change in slope (rotation) of the curve over the entire range of input signal level must be less than 1.25:1. This corresponds to a change in signal level after limiting of 2 db.

For a finite IF and a discriminator, amplifiers A_1 and A_2 will be assumed to be single-tuned stages. The gain of A_1 normalized to unity at its center frequency is

$$K_{in} = \frac{1}{\sqrt{1 + \left(\frac{f^2 - f_1^2}{f B_1}\right)}} \qquad (36)$$

where f_1 is the center frequency for A_1 and B_1 is the frequency difference between the 3 db points of A_1. A similar expression may be written for K_{2n}. The output of the frequency-to-voltage converter for this system is of the form

$$e_c = K e_{in} \frac{1}{\sqrt{1 + \left(\frac{f^2 - f_1^2}{f B_1}\right)^2}} \; \frac{1}{\sqrt{1 + \left(\frac{f_2^2 - f^2}{f B_2}\right)^2}} \qquad (37)$$

Taking B_1 and B_2 at 20 megacycles, f_1 at 48 megacycles, and f_2 as 70 megacycles gives an offset frequency f_c of 60 megacycles and a fairly linear transfer function curve, e_c to x for the values of x between -7.5 and +7.5 megacycles. ($x = f - f_c$). A plot of the curve normalized to e_c equal to unity at x equal to -7.5 megacycles is shown in Figure 10.

The ratio of $\gamma_{min} / \gamma_{max}$ for this curve is 0.9. Thus, the change in slope due to imperfect limiting must be less than 3 db in order that the specification on tracking capability be met with an AFC bandwidth of 15 Mc.

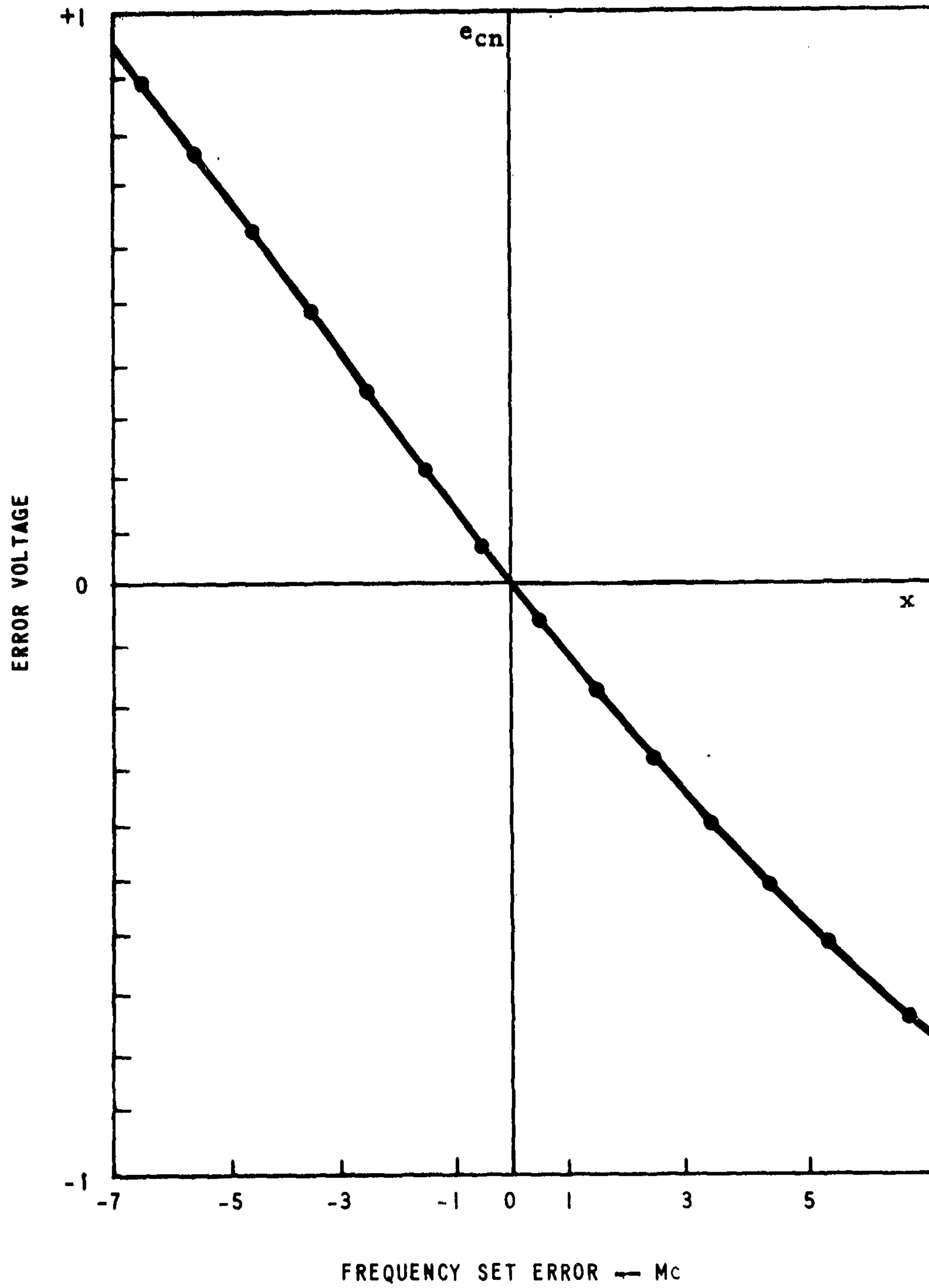

Figure 9

Error Voltage versus Frequency Set Error for
a Slope Detector with a Constant Amplitude Input

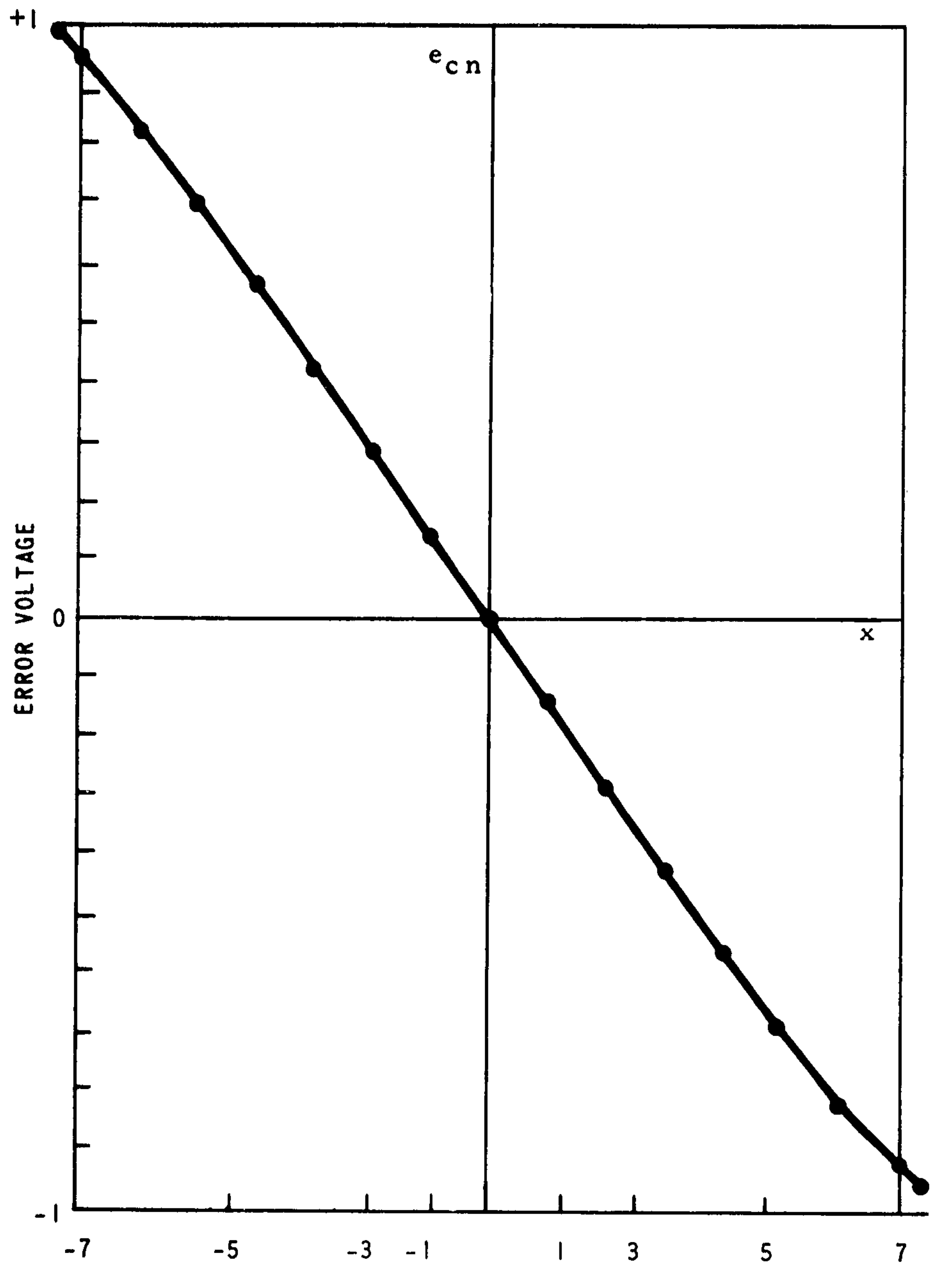

Figure 10

Error Voltage versus Frequency Set Error for
a Discriminator with a Constant Amplitude Input

6.7 -- Continued.

With the AFC bandwidth fixed, the maximum search rate for 100 per cent probability of intercept may be calculated.

$$\left|\frac{d f_o}{d t}\right|_{max} = \frac{15 \text{ Mc}}{.01 \text{ sec}} = 1.5 \text{ Gc/sec} \tag{38}$$

An estimate of the maximum attainable sensitivity may be made starting with a nominal specification of a false alarm rate of less than one false alarm per second. For the zero IF system, Equation 90 may be solved for the required signal to noise ratio, D, at the output of the amplifier A_o:

$$D > 16 \approx 12 \text{ db} \tag{39}$$

Note that this definition of signal to noise is the ratio of mean square noise voltage, in the absence of signal, to mean square signal voltage at zero frequency set error.

Estimating the noise figure of the mixer, local oscillator, and amplifier A_o as 10 db, we have from Equation 31

$$S \geq -114 + 16 + 12 + 10 = -76 \text{ dbm} \tag{40}$$

The maximum sensitivity of the finite IF system will be slightly different since the noise bandwidth is 15 megacycles as compared to 20 megacycles for the zero IF system. For the finite IF system

$$S \geq -114 + 15 + 12 + 10 = -77 \text{ dbm} \tag{41}$$

Since the specification on sensitivity is more than adequately satisfied in both cases, some of the component performance requirements could be relaxed slightly. For example, a slightly greater AFC bandwidth could be used so that the limiting requirements could be reduced while maintaining the same tracking capability. On the other hand, if the sensitivity specification had been 80 dbm, either the tracking capability would have to be reduced or the LO and mixer noise figure improved.

7. SUMMARY AND CONCLUSIONS.

The AFC system considered in this report has been reduced to four basic blocks. A transfer function, not necessarily linear, has been ascribed to each of these blocks. The system behavior has been described in general terms. The particular components making up these blocks are important only insofar as they determine the linearity of these transfer functions and govern the degree to which these transfer functions change with different inputs.

The problem has been approached from the point of view of extracting the most useful design information. This has permitted analytical inclusion of nonlinear effects, at least to the extent of limit points (maxima and minima). These are really the dominant design considerations. For example, it is not sufficient for a system to be stable under average operating conditions; it must be stable under all specified conditions.

Extension of the concepts outlined in this report, to describe the systems dynamic behavior under a given set of signal conditions, would not be difficult. Analysis of an existing system for which the transfer functions are known could be made quite simply using graphical techniques. A complete analysis of an unspecified system would require piecewise linear approximations of the transfer functions.

No reference is made in the report to any particular active electronic component. The choice between transistors and vacuum tubes will, of course, depend on specifications such as size, weight, and efficiency and the ability to perform the required functions. It is felt that the material presented here will prove valuable as background material in selecting a particular system configuration, as well as in the design once the choice is made.

8. APPENDIX.

8.1 Frequency-to-Voltage Converter.

The function of the frequency-to-voltage converter is to provide pulses whose amplitudes are directly proportional to the instantaneous frequency error, $f_o - f_s - f_c$. An ideal circuit would maintain the same relationship between output pulse amplitude and frequency regardless of the amplitude or duration of the input pulse. When the input pulse amplitudes may vary over a wide range and when very narrow pulses

8.1 -- Continued.

may be received, this ideal behavior will be difficult to achieve. Some variation in the slope of the output pulse amplitude versus frequency error curve may be tolerated at the expense of tracking capability, but the existence of a unique output amplitude at zero frequency error is of paramount importance.

8.1.1 A Frequency-to-Voltage Converter Using a Finite IF and a Discriminator. In the diagram of Figure 11, the block (A_0) is a bandpass amplifier with center frequency f_c. Simple diode limiting will be used in this amplifier to reduce the dynamic range to a reasonable value (say 3 db variation in output amplitude).

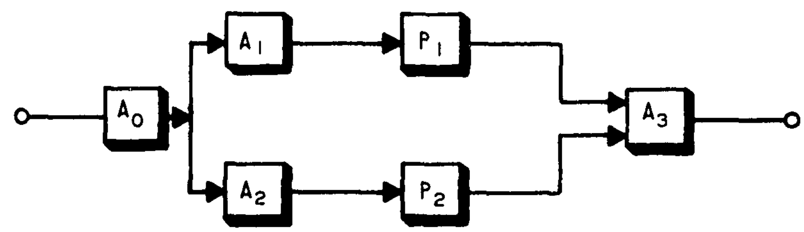

Figure 11

Frequency-to-Voltage Converter

Amplifier A_1 is tuned to a center frequency f_1, which is lower than f_c. The output amplitude of A_1 is then a decreasing function of frequency for $f > f_1$. Similarly, A_2 is tuned to a center frequency f_2, which is greater than f_c, so that its output is an increasing function of frequency for $f < f_2$. The outputs of A_1 and A_2 are detected and the peak amplitudes are stretched in P_1 and P_2. The outputs of P_1 and P_2 are then long pulses with amplitudes corresponding to the maximum amplitudes out of A_1 and A_2 respectively. These pulses are subtracted in the difference amplifier, A_3, to give the discriminator characteristic of Figure 12. This discriminator configuration

8.1.1 -- Continued.

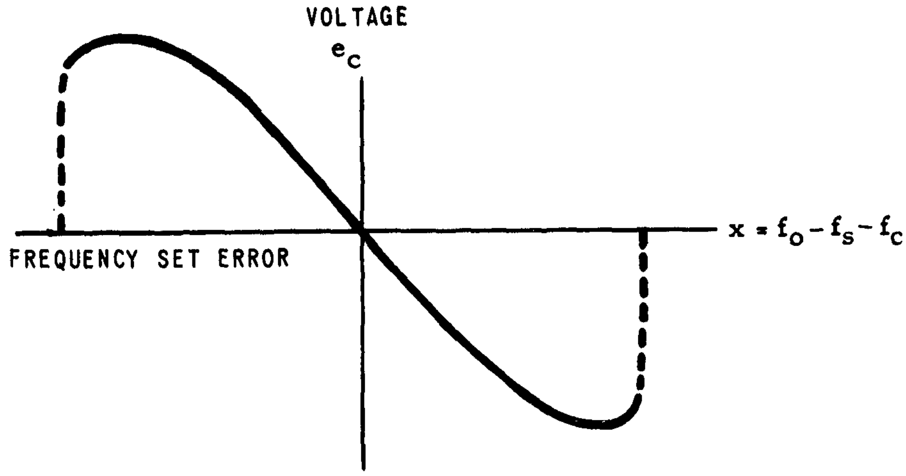

Figure 12

Discriminator Curve

is chosen to have separate noninteracting information channels out to the stretched pulse level. This permits independent adjustments of amplifiers A_1 and A_2 to match gains and rise times. Also, subtraction is accomplished with long pulses, hence the frequency response of the subtraction circuit may be low.

Obviously, theoretical determination of the curve of Figure 12 for pulsed signals depends on a transient analysis of the circuit of Figure 11. The only meaningful steady-state conditions that could be defined here would be for steady-state behavior within the duration of a single pulse. A complete analysis of this circuit, taking into account the finite rise time of the input pulse, amplitude variations of the input pulse, and rise time differences for A_1 and A_2, is difficult, if not impossible. The procedure used here will be to start with a quasi-steady-state analysis, bringing in the departures from ideal behavior as perturbations. Specifically, the equations for steady-state behavior will be used for all cases. It will be assumed that the effect of incomplete rise for narrow pulses may be considered as a diminution of the mid-band gain.

8.1.1 -- Continued.

In the analysis to follow, A_1 and A_2 will be assumed to be single-tuned stages. It may be simply shown that for the steady state, the peak values of the outputs of A_1 and A_2 are:

$$e_{o1} = \frac{K_o K_1 e_i}{\sqrt{1 + \left(\frac{f^2 - f_1^2}{B_1 f}\right)^2}} , \tag{42}$$

$$e_{o2} = \frac{K_o K_2 e_i}{\sqrt{1 + \left(\frac{f_2^2 - f^2}{B_2 f}\right)^2}} , \tag{43}$$

where K_1 and K_2 are the amplifier gains of A_1 and A_2, $K_o e_i$ is the peak output of A_o, B_1 is the difference between the 3-db frequencies for A_1, and B_2 is defined in a similar fashion. The output pulse amplitude is given by

$$e_c = K_3 (e_{o1} - e_{o2}) , \tag{44}$$

where K_3 is the difference amplifier gain. Since the response curves of A_1 and A_2 are not completely symmetrical about their center frequencies, the values of K_1 and B_1 may be made slightly different from the corresponding quantities in A_2 in order to get the most symmetrical composite characteristic. However, these differences will be slight and will have little effect on the calculations to follow. Taking K_1 equal to K_2 and B_1 equal to B_2

$$e_c = K_o K_1 K_3 e_i \left\{ \frac{1}{\sqrt{1 + \left(\frac{f^2 - f_1^2}{B_1 f}\right)^2}} - \frac{1}{\sqrt{1 + \left(\frac{f_2^2 - f^2}{B_1 f}\right)^2}} \right\} , \tag{45}$$

e_c will be zero for

$$f = f_c = \sqrt{\frac{f_1^2 + f_2^2}{2}} , \tag{46}$$

8.1.1 -- Continued.

Now consider the change in zero crossover frequency when the gain of A_1 or A_2 changes by a factor of $1 + \delta$. This may be due to incomplete rise for short pulses. The zero crossover now occurs for a difference frequency f given by:

$$(1+\delta)^2 \left[1 + \left(\frac{f^2 - f_1^2}{B_1 f} \right)^2 \right] = 1 + \left(\frac{f_2^2 - f^2}{B_1 f} \right)^2, \qquad (47)$$

For

$$(\delta^2 + 2\delta) B_1^2 \ll 2 \left[f_2^2 - (1+\delta)^2 f_1^2 \right], ^* \qquad (48)$$

we have for the change in crossover frequency

$$\Delta f_c \approx \delta/4 \ (f_2 - f_1), \qquad (49)$$

To find the slope of the discriminator curve at the origin for the unperturbed case where $K_1 = K_2$, define

$$x = \frac{f_2^2 - f^2}{B_1 f}, \qquad (50)$$

$$y = \frac{f^2 - f_1^2}{B_1 f}, \qquad (51)$$

* For example, if $B_1 = 20$, $\delta = 0.1$, $f_1 = 48$, $f_2 = 70$, $2\left[f_2^2 - (1+\delta)^2 f_1^2\right] = 4225$; $(\delta^2 + 2\delta) B_1^2 = 84$.

+ For the same example: $\Delta f_c = 0.55$ Mc.

8.1.1 -- Continued.

$$\frac{d\,e_c}{d\,f} = K_o\ K_1\ K_3\ e_i \left\{ \frac{-y\ \frac{dy}{df}}{(1+y^2)^{3/2}} + \frac{x\ \frac{dx}{df}}{(1+x^2)^{3/2}} \right\} \tag{52}$$

Since $x = y$ for $f = f_c$, this expression reduces to

$$\gamma_o = \left.\frac{de_c}{df}\right|_{f=f_c} = \frac{-4\ K_o\ K_1\ K_3\ e_i}{B_1}\ \frac{k}{(1+k^2)^{3/2}} \tag{53}$$

where

$$k = \frac{f_2 - f_1}{B_1} \tag{54}$$

The expression for γ_o may be used to calculate static errors. For example, if a threshold switching voltage E must be overcome in order to achieve frequency correction, the corresponding frequency set uncertainty would be $E\gamma_o$.

8.1.2 A Frequency to Voltage Converter Using a Slope Detector.

The analysis here will be for a "Zero IF" system, i.e., amplifier A_o in Figure 11 will be a video amplifier. The output of amplifier A_1 will again be a decreasing function of frequency, but A_2 will be wide band, i.e., its gain will be independent of frequency over the range of interest. The other components in the diagram of Figure 11 will be the same as in the previous section.

Assuming that A_1 has a single pole roll-off:

$$e_{o1} = \frac{K_o\ K_1\ e_i}{\sqrt{1 + (f/B_1)^2}} \tag{55}$$

$$e_{o2} = K_o\ K_2\ e_i \tag{56}$$

8.1.2 -- Continued.

where B_1 is the upper 3 db frequency for A_1.

Defining

$$K_2 = K_1 \eta \ , \tag{57}$$

The expression for e_c is

$$e_c = K_o K_1 K_3 e_i \left\{ \frac{1}{\sqrt{1 + (f/B_1)^2}} - \frac{1}{\eta} \right\} \tag{58}$$

$e_c = 0$ for

$$f = f_c = B_1 \sqrt{\eta^2 - 1} \tag{59}$$

If the gain of A_1 or A_2 changes by a factor of $1 + \delta$, $e_c = 0$ for

$$f = B_1 \sqrt{(\delta + 1)^2 \ \eta^2 - 1} \tag{60}$$

or

$$\Delta f_c \approx \frac{\eta^2 \ \delta B_1}{\sqrt{\eta^2 - 1}} \tag{61}$$

This is a minimum for $\eta^2 = 2$.

Using this value of η^2

$$\Delta f_c = 2 \ \delta \ B_1 \ * \tag{62}$$

To find the slope of the slope detector curve at the origin for the unperturbed case with $\eta^2 = 2$

$$\gamma_o = \frac{de_c}{df} \bigg|_{f = f_c} = \frac{-f_c}{B_1^2} \left[1 + (f_c/B_1)^2 \right]^{-3/2} K_o K_1 K_3 e_i \tag{63}$$

* For example, with $\delta = 0.1$ and $B_1 = 12.5$ megacycles:
$\Delta f_c = 2.5$ megacycles.

8.1.2 -- Continued.

Thus, for $\eta^2 = 2$:

$$\gamma_o = \frac{de_c}{df}\Big|_{f = f_c} = \gamma_o = \frac{-K_o \; K_1 \; K_3 \; e_i}{2\sqrt{2}\; B_1} \tag{64}$$

8.2 Frequency Control Circuit.

In the absence of a signal or when the difference frequency ($f_o - f_s$) is outside the bandpass of the frequency-to-voltage converter, the function of the frequency control circuit is to tune the LO through the desired frequency range. This phase of the operation will be referred to as the search or open loop mode. When the difference frequency is within the bandpass of the frequency to voltage converter, the frequency control circuit will provide a net change in LO tuning voltage per pulse that is proportional to the frequency error $x = f_o - f_s - f_c$. This phase of the operation will be called the track or closed-loop mode.

8.2.1 Circuit Configuration. A circuit capable of performing the indicated operation is shown in Figure 13.

The dual search-track function is provided for by the electronic switches S_1 and S_2. With S_1 closed and S_2 open, the circuit configuration is that of the well known Miller integrator. (Amplifier A_6 has unity gain. Its only function is to prevent the current through C_p from flowing through the load resistor of A_4). If provision is made for re-cycling, the circuit is a free-running sweep generator (the self re-cycling phantastron circuit may be used). A_5 represents all stages required to achieve the necessary polarity and voltage swing to tune the LO through the desired frequency range.

With the first pulse from the frequency-to-voltage converter, S_1 is to be opened and held open as long as pulses are being received or until a sufficient time has elapsed to indicate loss of signal. This may be accomplished simply by using a diode or transistor switch that is held open by a voltage that is derived from the integral of the pulses from one of the pulse stretchers.

8.2.1 -- Continued.

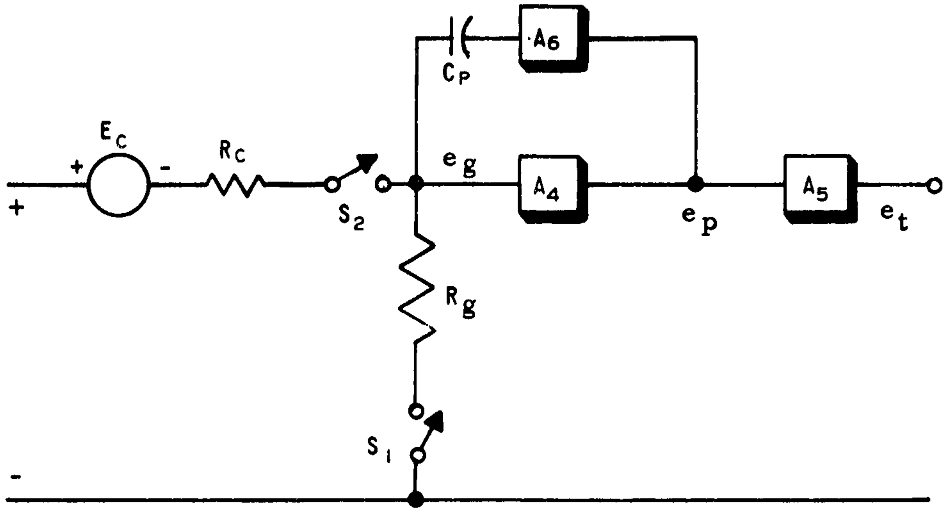

Figure 13

A Frequency Control Circuit

As mentioned in Section 3.2.4, e_p is not to vary in closed loop operation, except during a fixed correction interval T, provided with each pulse from the frequency-to-voltage converter. Referring to Figure 13, if switches S_1 and S_2 are ideal, e_p cannot change with both switches open. Thus, if S_2 is closed only for the interval T, the desired operation may be achieved. For the specific system to be discussed here, T will be equal to the width of the stretched pulses. Thus, S_2 may be operated by the pulses from one of the pulse stretchers. Realization of the switch S_2 will be discussed in Section 8.2.4.

Summarizing the operation of the switches for this circuit:

1. Open loop (search) mode: S_1 closed, S_2 open.

2. Closed loop (track) mode: S_1 open

   a. S_2 closed when $|e_c| > 0$

   b. S_2 open when $e_c = 0$.

8.2.2 Equations Governing the Search Mode. Analysis of the frequency control circuit with S_1 closed and S_2 open proceeds as follows:

If $-K_4$ is the gain of the amplifier A_4 *, the equation relating e_p to e_g is:

$$e_p - E_{po} = -K_4 \; (e_g + E_{go}) \tag{65}$$

where $e_{g(0^+)} = -E_{go}$ and $e_{p(0^+)} = E_{po}$.

The second equation required is obtained by equating the capacitor current to the current flowing through R_g.

$$\frac{e_g}{R_g} = C_p \frac{d}{pt} \; (e_p - e_g). \tag{66}$$

The Laplace transforms of these differential equations are:

$$K_4 e_g + e_p = \frac{E_{po} - K_4 \; E_{go}}{s} \tag{67}$$

$$-(s + \frac{1}{R_g C_p}) \; e_g + s \; e_p = E_{po} + E_{go}. \tag{68}$$

The solutions of this set of equations are:

$$e_p = E_{po} - K_4 \; E_{go} \; (1 - e^{-P_1 t}) \tag{69}$$

$$e_g = -E_{go} \; e^{-P_1 t} \tag{70}$$

* Since the total output swing may be of the order of the supply voltage used, taking K_4 as a constant is a very gross linearization. It will be shown that the circuit may be designed to make the effects of variation of K_4 unimportant.

8.2.2. -- Continued.

where

$$P_1 = \frac{1}{R_g C_p (K_4 + 1)} \tag{71}$$

For $P_1 t \ll 1$:

$$e^{-P_1 t} \approx 1 - P_1 t \tag{72}$$

and

$$e_p - E_{po} \approx \frac{-K_4}{K_4 + 1} E_{go} \frac{t}{R_g C_p} \tag{73}$$

$$e_g + E_{go} \approx \frac{E_{go}}{K_4 + 1} \frac{t}{R_g C_p} \tag{74}$$

If K_4 is constant, there are the equations of ramp functions. * Even if K_4 changes considerably during the sweep, the waveform of e_p will be quite linear as long as $K_4 \gg 1$.

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* If a more accurate representation is desired, one may divide the total output swing into n segments ($E_{p0} - E_{p1}$, $E_{p1} - E_{p2}$, ... $E_{pk-1} - E_{pk}$ . . . . $E_{p-n} - E_{pn}$), defining K_4 in each segment as K_{4k}, where k is an integer between 1 and n. The waveforms of e_p and e_g may then be calculated by starting with E_{po} and E_{go} and plotting these equations for $E_{po} \geq e_p \geq E_{p1}$, using the corresponding value of K_4 (K_{41}). The endpoints of this solution (E_{p1}, E_{g1}) define the initial conditions for the next plot using K_{42} and $E_{p1} \geq e_p \geq E_{p2}$. This process may be repeated until $e_p = E_{pn}$, and the complete time response is plotted.

8.2.3 Equations Governing Track Operation. In closed-loop operation, S_1 is open. When S_2 is open, e_c remains constant, so the only solution of interest is for S_2 closed with a correction pulse present ($|e_c| > 0$).

In general, one is interested here in a small signal solution, and the linearization of K_4 over a small change in output is quite valid. For example, if the bandwidth of the frequency-to-voltage converter is 20 megacycles, then the maximum error that will be encountered in closed-loop operation is 10 megacycles. Assuming that a total frequency range of 2 gigacycles is to be covered with a 50 volt change in e_p, then the variation in e_p necessary to correct for a 10-megacycle error is:

$$\Delta e_p = \frac{10}{2000} \times 50 = 1/4 \text{ volt.} \tag{75}$$

The small signal equations for the incremental variation in output voltage ($\tilde{e}_p$) and the voltage ($\tilde{e}_g$) are:

$$K_4 \tilde{e}_g + \tilde{e}_p = 0 \tag{76}$$

$$-\left(s + \frac{1}{R_c C_p}\right)\tilde{e}_g + s\tilde{e}_p = \frac{E_c - E_g - e_c}{s R_c C_p} \tag{77}$$

where e_c is the amplitude of the frequency correction pulse, E_c is a fixed bias added to that pulse, and $-E_g$ is the value of e_g at the start of the correction pulse.

The solution of these equations is:

$$\tilde{e}_p = -K_4 (e_c + E_g - E_c)\ (1 - e^{-P_1 t}) \tag{78}$$

where

$$P_1 = \frac{1}{(K_4 + 1)\ C_p R_c} \tag{79}$$

The frequency correction pulse will be of a fixed width T. For PT<< 1, the net change in e_p per pulse (Δe_p) will be:

8.2.3 - - Continued.

$$\Delta e_p \approx \frac{-K_4 \; T}{(1 + K_4)\, R_c \; C_p} \left\{ E_g - E_c + e_c \right\} \qquad (80)$$

For proportional correction, feedback from the output may be used to adjust the bias level E_c, so that $E_g - E_c$ is negligible. If $K_4 >> 1$,

$$\Delta e_p \approx \frac{-T}{R_c \; C_p} \; e_c \qquad (81)$$

If K_5 is the gain of the linear amplifier following the integrator:

$$\Delta e_t \approx \frac{-T \; K_5}{R_c \; C_p} \; e_c = \alpha \; e_c \qquad (82)$$

8.2.4 Switching Details. One method of realizing switch S_1 has already been suggested. Two biased diodes (D_1 and D_2) may be used to realize S_2, as shown in Figure 14.

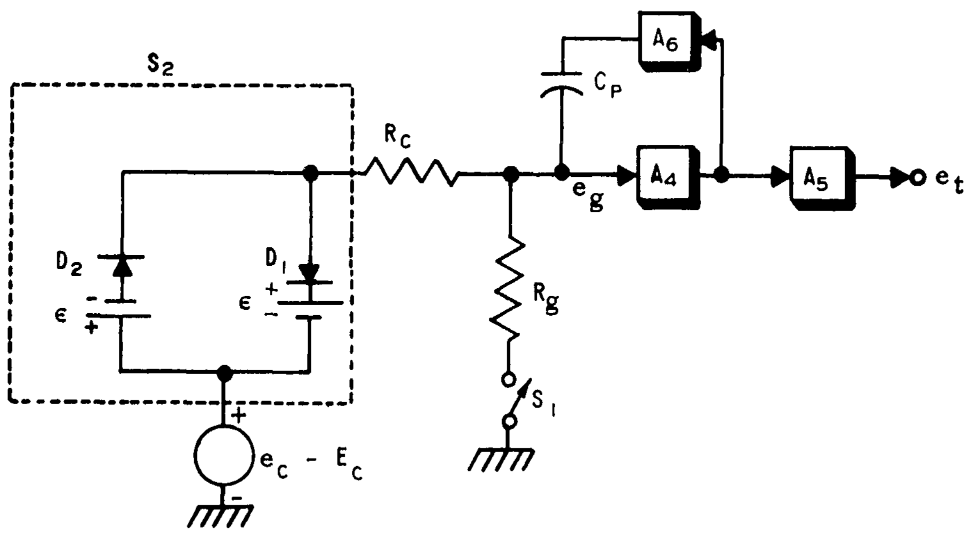

Figure 14

Frequency Control Schematic

8.2.4 -- Continued.

Recall that S_2 is to be open for $e_c = 0$, and closed for $| e_c | > 0$. The circuit above approximates these functions very closely. For $e_c = 0$ and

$$-\epsilon < e_g + E_c < \epsilon \tag{83}$$

D_1 and D_2 are both back biased. However, from the equations governing closed loop operation (Equation 80), $e_g + E_c = -(E_g - E_c)$ must be small in order that the change in output per pulse can be linearly related to e_c. Thus, if the circuit is designed to operate accurately in closed loop operation, ϵ may be quite small.

Now, when $| e_c | > 0$,

(a) if $e_c > 2 \epsilon$, D_2 is forward biased.

(b) if $e_c < -2 \epsilon$, D_1 is forward biased.

Thus, switch S_2 is closed for

$$| e_c | > 2 \epsilon \tag{84}$$

As $\epsilon_g - \epsilon_c$ approaches zero, ϵ may approach zero, and the ideal switching function may be closely approximated. 2ϵ represents the minimum magnitude of e_c for which frequency correction may be achieved. Dividing 2ϵ by the slope of the frequency-to-voltage converter transfer function curve at the origin (γ_o) gives a lower bound on the frequency set error.

If S_1 is an electronic switch (transistor or diode), a leakage current will exist which will cause a drift in e_t between pulses. For a constant leakage current i_L, the change in e_t during the interval between pulses (τ) will be

$$\Delta e_t = \frac{K_5 \; i_L \, \tau}{e_p} \tag{85}$$

and the corresponding peak error due to the change in LO frequency will be

$$\Delta x = \frac{\beta \; K_5 \; i_L \; \tau}{c_p} \tag{86}$$

8.3 The Effects of Noise.

The nonlinear behavior of several of the circuits in the frequency-to-voltage converter makes the prediction of the effect of noise on the output pulses extremely difficult. For example, amplifier A_o contains a limiter whose behavior is decidedly nonlinear. There is evidence to indicate that the signal to noise ratio will not be altered significantly (and perhaps even enhanced slightly) by the limiter. [3] However, calculation of the effect of noise on the outputs of the pulse stretchers and ultimately on the output pulse which is produced by subtracting these pulses in the difference amplifier was not attempted and no references have been found which consider this particular problem.

Fortunately, an estimate of the system false alarm rate can be made which often obviates the necessity of making the aforementioned calculations. In several typical AFC systems tested, the contribution of noise to the frequency set error was negligible when the signal to noise ratio was high enough to give a tolerable false alarm rate.

8.3.1 False Alarm Rate.

Figure 15 is a block diagram of the frequency to voltage converter. The gate generator used to control the widths of the stretched pulses is shown explicitly in this diagram.

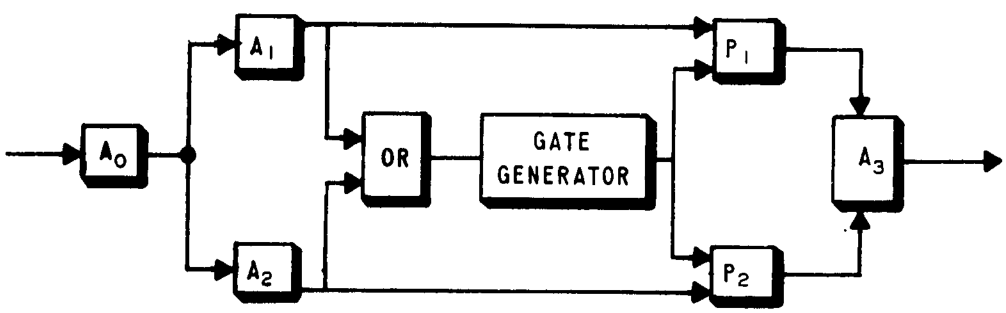

Figure 15

Frequency-to-Voltage Converter

The function of this gate generator is to enable pulse stretchers P_1 and P_2 to store the peak amplitudes of the signal pulses out of A_1 and A_2 for a prescribed interval (equal to the width of the gate pulse), after which the stretched pulses are terminated. The output of the gate generator may also be used to switch the frequency control circuit from open loop to closed loop operation. A false alarm will thus be defined as any spurious triggering of the gate generator.

8.3.1 -- Continued.

In the discriminator configuration, the gate generator may be triggered by a pulse from either channel, as shown in Figure 15. In the slope detector configuration, the gate generator will be triggered from the wide band amplifier A_2. In this way, the critical amplitude for triggering (e_1) may be set at slightly less than the peak signal pulse amplitude out of A_1 or A_2 at zero frequency set error (e_o) (the outputs of A_1 and A_2 are equal at zero error).

Following the work of Rice [4], the number of times per second that the noise voltage can be expected to pass through the value of e_1 with a positive slope is:

$$n = \frac{1}{2} N_z e^{-\frac{e_1^2}{2\sigma^2}} \qquad (87)$$

where N_z is the number of expected zeros per second and σ^2 is the mean-square noise voltage which is given by:

$$\sigma^2 = \int_0^{\infty} S_{(f)}\, df. \qquad (88)$$

Here $S_{(f)}$ is the power spectral density of the noise out of the amplifier in question. The formula for N_z is:

$$N_z = 2 \left[\frac{\int_0^{\infty} f^2 S_{(f)}\, df}{\int_0^{\infty} S_{(f)}\, df} \right]^{1/2} \qquad (89)$$

If amplifier A_o is assumed to be an ideal video amplifier and the input noise is assumed to be white, an estimate of the false alarm rate for the slope detector configuration may be made quite simply. Assume that the wide band amplifier A_2 is an ideal video filter of bandwidth B_2 and that the noise spectral density is N_o (volts)^2 per megacycle. Then

$$S_{(f)} = \begin{cases} K N_o & 0 < f \leq B_2 \\ 0 & B_2 < f \end{cases} \qquad (90)$$

where K is the total gain of amplifiers A_o and A_2. Substituting 88 into 87 and integrating:

$$N_z = \frac{2B_2}{\sqrt{3}} \qquad (91)$$

8.3.1 -- Continued.

Taking e_1 equal to e_o,

$$n = \frac{B_2}{\sqrt{3}} e^{-D} \tag{92}$$

where

$$D = e_o^{\,2}/2\sigma^2 .$$

$\frac{e_o^{\,2}}{2}$ is the mean square signal voltage out of A_2 at zero frequency error. Thus, $e_o^{\,2}/2\sigma^2$ is the ratio of mean square signal voltage at zero frequency error to mean square noise voltage in the absence of a signal.

We see that n is a very strong function of this signal to noise ratio. For example, if $B_2 = 20 \times 10^6$ cps and n is specified to be less than one false alarm per second:

$$D = \frac{e_o^{\,2}}{2\sigma^2} > 16.3 \approx 12.1 \text{ db.} \tag{93}$$

For D = 13 db, n = .024 sec. By contrast, for D = 11 db, n = 38.4 per sec.

Similar calculations may be made for the discriminator by approximating A_1 and A_2 as Gaussian filters. A more simple (but less accurate) estimate may be made by replacing the composite filter consisting of A_1 and A_2 by a single ideal bandpass filter of bandwidth B. This is equivalent to providing a separate triggering channel for the gate generator. In this case:

$$S_{(f)} = \begin{cases} K N_o & f_a < f < f_b \\ 0 & \text{elsewhere} \end{cases} \tag{94}$$

Then,

$$N_z = 2 \left[\frac{(f_a + f_b)^2 - f_a f_b}{3} \right]^{1/2} \tag{95}$$

Setting

$$f_b \approx f_c + \frac{B}{2} \tag{96}$$

8.3.1 -- Continued.

and

$$f_a \approx f_c - \frac{B}{2} . \qquad (97)$$

$$Nz = 2 \left(f_c^2 - B^2/12 \right)^{1/2} \qquad (98)$$

which is within 1 per cent of $2 f_c$ for $B < f_c/2$. Taking $Nz = 2 f_c$:

$$N = f_c e^{-D} \qquad (99)$$

For fc 6×10^7 cps and $N > 1$:

$$D > 17.9 = 12.5 \text{ db}. \qquad (100)$$

9. REFERENCES.

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4. S. O. Rice, "Mathematical Analysis of Random Noise," B.S.T.J., v 23, p 282, July 1944; and v 24, p 46, January 1945.

AD______________ Accession No. ________ UNCLASSIFIED Copy No.

Electronic Defense Labs., Mountain View, Calif. ANALYSIS OF AN AFC SYSTEM FOR AN AUTOMATIC SEARCH-TRACK RECEIVER - Harold E. Thiel and Keith E. Root. Technical Memorandum EDL-M443, 28 February 1962 (Contract DA 36-039 SC-87475) UNCLASSIFIED Report.

The general behavior of a sampled data automatic frequency control loop for use in receiving pulsed radio frequency signals is described. A functional description of the system blocks is used to develop automatic frequency control (AFC) loop stability criteria. Tracking capabilities of the AFC loop are formulated and equations are derived to permit an estimation of frequency set errors. Design considerations are discussed. The frequency to voltage converter and the frequency control circuit are discussed in some detail in the appendixes.

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